A Study on the Use of Shehu Transform for Differential Equation Modeling

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Affliction: Renal Function Decline Syndrome

ABSTRACT

In recent years, many scholars have paid attention to find the solution of advance problems of engineering and sciences by using integral transforms method. In this paper, application of Shehu transform is given for handling growth and decay problems. These problems have much importance in the field of economics, chemistry, biology, physics, social science and zoology. We have given some numerical applications to demonstrate the effectiveness of Shehu transform for handling growth and decay problems. Results prove that Shehu transform is quite useful for handling growth and decay problems.

Keywords: *Shehu transform, Inverse Shehu transform, Growth and Decay problems, Half-life.*

I. INTRODUCTION

Integral transforms methods (Laplace transform [1-2], Fourier transform [1], Kamal transform [3-9, 36], Mahgoub transform [10-16], Mohand transform [17-20, 37-40], Aboodh transform [21-26, 41-44], Elzaki transform [27-29, 45-45], Sumudu transform [30, 47-48] and Shehu transform [49]) are convenient mathematical methods for solving advance problems of sciences and engineering which are defined in terms of differential equations, system of differential equations, partial differential equations, integral equations, system of integral equations, partial integro-differential equations and integro differential equations. Aggarwal et al. [31-35] discussed the comparative study of Mohand and other transforms.

Shehu transform of the function $F(t), t \geq 0$ is given by [49]:

$S\{F(t)\} = \int_0^\infty F(t)e^{-\frac{vt}{u}}dt = H(v,u), v > 0, u > 0,$ where operator S is called the Shehu transform operator.

The Shehu transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. The mention two conditions are the only sufficient conditions for the existence of Shehu transforms of the function $F(t)$.

The growth of a plant, or a cell, or an organ, or a species is mathematically expressed in terms of a first order ordinary linear differential equation [50-54] as

$$\frac{dQ}{dt} = KQ \quad (1)$$

with initial condition $Q(t_0) = Q_0$ (2)

where K is a positive real number, Q is the amount of population at time t and Q_0 is the initial population at time $t = t_0$.

Equation (1) is known as the Malthusian law of population growth.

The decay problem of the substance is defined mathematically by the following first order ordinary linear differential equation [51, 54] as

$$\frac{dQ}{dt} = -KQ \quad (3)$$

with initial condition $Q(t_0) = Q_0$ (4)

where Q is the amount of substance at time t, K is a positive real number and Q_0 is the initial amount of the substance at time $t = t_0$.

In equation (3), the negative sign in the R.H.S. is taken because the mass of the substance is decreasing with time and so the derivative $\frac{dQ}{dt}$ must be negative.

In this paper, application of Shehu transform is given for handling growth and decay problems.

II. LINEARITY PROPERTY OF SHEHU TRANSFORMS

If $S\{F(t)\} = H_1(v,u)$ and $S\{G(t)\} = H_2(v,u)$ then $S\{aF(t) + bG(t)\} = aS\{F(t)\} + bS\{G(t)\}$
$\Rightarrow S\{aF(t) + bG(t)\} = a\,H_1(v,u) + bH_2(v,u)$, where a, b are arbitrary constants.

III. SHEHU TRANSFORM OF SOME USEFUL FUNCTIONS

S.N.	$F(t)$	$S\{F(t)\} = H(v,u)$
1.	1	$\frac{u}{v}$
2.	t	$\left(\frac{u}{v}\right)^2$
3.	t^2	$2!\left(\frac{u}{v}\right)^3$
4.	$t^n, n \in N$	$n!\left(\frac{u}{v}\right)^{n+1}$
5.	$t^n, n > -1$	$\Gamma(n+1)\left(\frac{u}{v}\right)^{n+1}$
6.	e^{at}	$\frac{u}{v-au}$
7.	$sinat$	$\frac{au^2}{(v^2+a^2u^2)}$
8.	$cosat$	$\frac{uv}{(v^2+a^2u^2)}$
9.	$sinhat$	$\frac{au^2}{(v^2-a^2u^2)}$
10.	$coshat$	$\frac{uv}{(v^2-a^2u^2)}$
11	$J_0(t)$	$\frac{u}{\sqrt{(v^2+a^2u^2)}}$

IV. INVERSE SHEHU TRANSFORM

If $S\{F(t)\} = H(v,u)$ then $F(t)$ is called the inverse Shehu transform of $H(v,u)$and mathematically, it is defined as $F(t) = S^{-1}\{H(v,u)\}$, where the operator S^{-1} is called the inverse Shehu transform operator.

V. LINEARITY PROPERTY OF INVERSE SHEHU TRANSFORMS

If $S^{-1}\{H_1(v,u)\} = F(t)$ and $S^{-1}\{H_2(v,u)\} = G(t)$ then
$S^{-1}\{a\,H_1(v,u) + b\,H_2(v,u)\} = aS^{-1}\{H_1(v,u)\} + bS^{-1}\{H_2(v,u)\}$
$\Rightarrow S^{-1}\{a\,H_1(v,u) + b\,H_2(v,u)\} = aF(t) + bG(t)$, where a, b are arbitrary constants.

VI. INVERSE SHEHU TRANSFORM OF SOME USEFUL FUNCTIONS

S.N.	$H(v,u)$	$F(t) = S^{-1}\{H(v,u)\}$
1.	$\frac{u}{v}$	1

2.	$\left(\frac{u}{v}\right)^2$	t
3.	$\left(\frac{u}{v}\right)^3$	$\frac{t^2}{2!}$
4.	$\left(\frac{u}{v}\right)^{n+1}, n \in N$	$\frac{t^n}{n!}$
5.	$\left(\frac{u}{v}\right)^{n+1}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{u}{v - au}$	e^{at}
7.	$\frac{u^2}{(v^2 + a^2u^2)}$	$\frac{sinat}{a}$
8.	$\frac{uv}{(v^2 + a^2u^2)}$	$cosat$
9.	$\frac{u^2}{(v^2 - a^2u^2)}$	$\frac{sinhat}{a}$
10.	$\frac{uv}{(v^2 - a^2u^2)}$	$coshat$
11.	$\frac{u}{\sqrt{(v^2 + a^2u^2)}}$	$J_0(t)$

VII. SHEHU TRANSFORM OF THE DERIVATIVES OF THE FUNCTION $F(t)$ [49]:

If $S\{F(t)\} = H(v,u)$ then

a) $S\{F'(t)\} = \frac{v}{u}H(v,u) - F(0)$

b) $S\{F''(t)\} = \frac{v^2}{u^2}H(v,u) - \frac{v}{u}F(0) - F'(0)$

c) $S\{F^{(n)}(t)\} = \frac{v^n}{u^n}H(v,u) - \sum_{k=0}^{n-1}\left(\frac{v}{u}\right)^{n-(k+1)} F^{(k)}(0)$

VIII. SHEHU TRANSFORM FOR HANDLING GROWTH PROBLEM:

In this section, we present Shehu transform for handling growth problem given by (1) and (2). Taking Shehu transform on both sides of (1), we have

$$S\left\{\frac{dQ}{dt}\right\} = KS\{Q(t)\} \tag{5}$$

Now applying the property, Shehu transform of derivative of function, on (5), we have

$$\frac{v}{u}S\{Q(t)\} - Q(0) = KS\{Q(t)\} \tag{6}$$

Using (2) in (6) and on simplification, we have

$$\left(\frac{v}{u} - K\right) S\{Q(t)\} = Q_0$$

$$\Rightarrow S\{Q(t)\} = \frac{Q_0 u}{(v - Ku)} \tag{7}$$

Operating inverse Shehu transform on both sides of (7), we have

$$Q(t) = S^{-1}\left\{\frac{Q_0 u}{(v - Ku)}\right\}$$

$$\Rightarrow Q(t) = Q_0 S^{-1}\left\{\frac{u}{(v - Ku)}\right\}$$

$$\Rightarrow Q(t) = Q_0 e^{Kt} \tag{8}$$

which is the required amount of the population at time t.

IX. SHEHU TRANSFORM FOR HANDLING DECAY PROBLEM:

In this section, we present Shehu transform for handling decay problem which is mathematically expressed in terms of (3) and (4).
Applying the Shehu transform on both sides of (3), we have

$$S\left\{\frac{dQ}{dt}\right\} = -KS\{Q(t)\} \quad (9)$$

Now applying the property, Shehu transform of derivative of function, on (9), we have

$$\frac{v}{u}S\{Q(t)\} - Q(0) = -KS\{Q(t)\} \quad (10)$$

Using (4) in (10) and on simplification, we have

$$\left(\frac{v}{u} + K\right) S\{Q(t)\} = Q_0$$

$$\Rightarrow S\{Q(t)\} = \frac{Q_0 u}{(v+Ku)} \quad (11)$$

Operating inverse Shehu transform on both sides of (11), we have

$$Q(t) = S^{-1}\left\{\frac{Q_0 u}{(v + Ku)}\right\}$$

$$\Rightarrow Q(t) = Q_0 S^{-1}\left\{\frac{u}{(v + Ku)}\right\}$$

$$\Rightarrow Q(t) = Q_0 e^{-Kt} \quad (12)$$

which is the required amount of substance at time t.

X. APPLICATIONS

In this section, some applications are given in order to demonstrate the effectiveness of Shehu transform for solving growth and decay problems.

Application: 1 The population of a city grows at a rate proportional to the number of people presently living in the city. If after four years, the population has tripled, and after five years the population is 50,000, estimate the number of people initially living in the city.

This problem can be written in mathematical form as:

$$\frac{dQ(t)}{dt} = KQ(t) \quad (13)$$

where Q denote the number of people living in the city at any time t and K is the constant of proportionality. Consider Q_0 is the number of people initially living in the city at $t = 0$.
Applying the Shehu transform on both sides of (13), we have

$$S\left\{\frac{dQ}{dt}\right\} = KS\{Q(t)\} \quad (14)$$

Now applying the property, Shehu transform of derivative of function, on (14), we have

$$\frac{v}{u}S\{Q(t)\} - Q(0) = KS\{Q(t)\} \quad (15)$$

Since at $t = 0, Q = Q_0$, so using this in (15), we have

$$\left(\frac{v}{u} - K\right) S\{Q(t)\} = Q_0$$

$$\Rightarrow S\{Q(t)\} = \frac{Q_0 u}{(v-Ku)} \quad (16)$$

Operating inverse Shehu transform on both sides of (16), we have

$$Q(t) = S^{-1}\left\{\frac{Q_0 u}{(v - Ku)}\right\}$$

$\Rightarrow Q(t) = Q_0 S^{-1}\left\{\frac{u}{(v-Ku)}\right\}$
$\Rightarrow Q(t) = Q_0 e^{Kt}$ (17)

Now at $t = 4$, $Q = 3Q_0$, so using this in (17), we have
$3Q_0 = Q_0 e^{4K}$
$\Rightarrow e^{4K} = 3$
$\Rightarrow K = \frac{1}{4} log_e 3 = 0.275$ (18)

Now using the condition at $= 5$, $Q = 50{,}000$, in (17), we have
$50{,}000 = Q_0 e^{5K}$ (19)

Putting the value of K from (18) in (19), we have
$50{,}000 = Q_0 e^{5\times 0.275}$
$\Rightarrow 50{,}000 = 3.955 Q_0$
$\Rightarrow Q_0 \simeq 12642$ (20)

which are the required number of people initially living in the city.

Application: 2 A radioactive substance is known to decay at a rate proportional to the amount present. If initially there is 100 milligrams of the radioactive substance present and after six hours it is observed that the radioactive substance has lost 30 percent of its original mass, find the half life of the radioactive substance.
This problem can be written in mathematical form as:
$\frac{dQ(t)}{dt} = -KQ(t)$ (21)

where Q denote the amount of radioactive substance at time t and K is the constant of proportionality. Consider Q_0 is the initial amount of the radioactive substance at time $t = 0$.
Applying the Shehu transform on both sides of (21), we have
$S\left\{\frac{dQ}{dt}\right\} = -KS\{Q(t)\}$ (22)

Now applying the property, Shehu transform of derivative of function, on (22), we have
$\frac{v}{u} S\{Q(t)\} - Q(0) = -KS\{Q(t)\}$ (23)

Since at $t = 0, Q = Q_0 = 100$, so using this in (23), we have
$\frac{v}{u} S\{Q(t)\} - 100 = -KS\{Q(t)\}$
$\Rightarrow \left(\frac{v}{u} + K\right) S\{Q(t)\} = 100$
$\Rightarrow S\{Q(t)\} = \frac{100u}{(v+Ku)}$ (24)

Operating inverse Shehu transform on both sides of (24), we have
$Q(t) = S^{-1}\left\{\frac{100u}{(v+Ku)}\right\}$
$= 100S^{-1}\left\{\frac{u}{(v+Ku)}\right\}$
$\Rightarrow Q(t) = 100e^{-Kt}$ (25)

Now at $t = 6$, the radioactive substance has lost 30 percent of its original mass 100 mg so $Q = 100 - 30 = 70$, using this in (25), we have
$70 = 100e^{-6K}$
$\Rightarrow e^{-6K} = 0.70$
$\Rightarrow K = -\frac{1}{6} log_e 0.70 = 0.059$ (26)

We required t when $Q = \frac{Q_0}{2} = \frac{100}{2} = 50$ so from (25), we have
$$50 = 100e^{-Kt} \tag{27}$$

Putting the value of K from (26) in (27), we have
$$50 = 100e^{-0.059t}$$
$$\Rightarrow e^{-0.059t} = 0.50$$
$$\Rightarrow t = -\frac{1}{0.059} log_e 0.50$$
$$\Rightarrow t = 11.75 \text{ hours} \tag{28}$$

which is the required half-time of the radioactive substance.

XI. CONCLUSION

In this paper, we have successfully discussed the application of Shehu transform for handling growth and decay problems. The given numerical applications in application section show the importance of Shehu transform for handling growth and decay problems. In the future, Shehu transform can be used for solving other advance problems of science and engineering like heat conduction problem, vibration problems of beam and bar, electric circuit problem and mixtures problem.

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