# A Study on Analogical Reasoning and Human Problem-Solving Mechanisms

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Abstract. Logical and analogical reasoning are sometimes viewed as mutually exclusive alternatives, but formal logic is actually a highly con-strained and stylized method of using analogies. Before any subject can be formalized to the stage where logic can be applied to it, analogies must be used to derive an abstract representation from a mass of irrel-evant detail. After the formalization is complete, every logical step – of deduction, induction, or abduction – involves the application of some version of analogy. This paper analyzes the relationships between logical and analogical reasoning, and describes a highly efficient analogy engine that uses conceptual graphs as the knowledge representation. The same operations used to process analogies can be combined with Peirce's rules of inference to support an inference engine. Those operations, called the *canonical formation rules* for conceptual graphs, are widely used in CG systems for language understanding and scene recognition as well as anal-ogy finding and theorem proving. The same algorithms used to optimize analogy finding can be used to speed up all the methods of reasoning based on the canonical formation rules.

## 1 Analogy and Perception

Before discussing the use of analogy in reasoning, it is important to analyze the concept of analogy and its relationship to other cognitive processes. General-purpose dictionaries are usually a good starting point for conceptual analysis, but they seldom go into sufficient depth to resolve subtle distinctions. A typical dictionary lists synonyms for the word *analogy*, such as *similarity*, *resemblance*, and *correspondence*. Then it adds more specialized word senses, such as *a simi-larity in some respects of things that are otherwise dissimilar*, *a comparison that determines the degree of similarity*, or *an inference based on resemblance or cor-respondence*. In AI, analogy-finding programs have been written since the 1960s, but they often use definitions of analogy that are specialized to a particular application.

The VivoMind Analogy Engine (VAE), which is described in Section 3, is general enough to be used in any application domain. Therefore, VAE leads to fundamental questions about the nature of analogy that have been debated in the literature of cognitive science. One three-party debate has addressed many of those issues:

1. *Thesis*: For the Structure Mapping Engine (SME), Falkenheimer, Forbus, and Gentner (1989) defined analogy as the recognition that "one thing is

like another" if there is a mapping from a conceptual structure that de-scribes the first one to a conceptual structure that describes the second. Their implementation in SME has been applied to a wide variety of practi-cal applications and to psychological studies that compare the SME approach to the way people address the same problems.

2. *Antithesis*: In their critique of SME, Chalmers, French, and Hofstadter (1992) consider analogy to be an aspect of a more general cognitive function called *high-level perception* (HLP), by which an organism constructs a conceptual representation of a situation. They "argue that perceptual processes cannot be separated from other cognitive processes even in principle, and therefore that traditional artificial-intelligence models cannot be defended by suppos-ing the existence of a 'representation module' that supplies representations ready-made." They criticize the "hand-coded rigid representations" of SME and insist that "content-dependent, easily adaptable representations" must be "an essential part of any accurate model of cognition."

3. *Synthesis*: In summarizing the debate, Morrison and Dietrich (1995) ob-served that the two positions represent different perspectives on related, but different aspects of cognition: SME employs structure mapping as "a gen-eral mechanism for all kinds of possible comparison domains" while "HLP views analogy as a process from the bottom up; as a representation-building process based on low-level perceptual processes interacting with high-level concepts." In their response to the critics, Forbus et al. (1998) admitted that a greater integration with perceptual mechanisms is desirable, but they re-peated their claim that psychological evidence is "overwhelmingly" in favor of structure mapping "as a model of human analogical processing."

The VAE approach supports Point #3: a comprehensive theory of cognition must integrate the structure-building processes of perception with the structure-mapping processes of analogy. For finding analogies, VAE uses a high-speed im-plementation of structure mapping, but its algorithms are based on low-level operations that are also used to build the structures. In the first implemen-tation, the conceptual structures were built during the process of parsing and interpreting natural language. More recently, the same low-level operations have been used to build conceptual structures from sensory data and from percept-like patterns used in scene recognition. VAE demonstrates that perception, language understanding, and structure mapping can be based on the same kinds of oper-ations.

This paper discusses the interrelationships between logical and analogical rea-soning, analyzes the underlying cognitive processes in terms of Peirce's semiotics and his classification of reasoning, and shows how those processes are supported by VAE. The same graph operations that support analogical reasoning can also be used to

support formal reasoning. Instead of being mutually exclusive, logical reasoning is just a more cultivated variety of analogical reasoning. For many purposes, especially in language understanding, the analogical processes provide greater flexibility than the more constrained and less adaptable variety used in logic. But since logical and analogical reasoning share a common basis, they can be effectively used in combination.

## 2 Logical and Analogical Reasoning

In developing formal logic, Aristotle took Greek mathematics as his model. Like his predecessors Socrates and Plato, Aristotle was impressed with the rigor and precision of geometrical proofs. His goal was to formalize and generalize those proof procedures and apply them to philosophy, science, and all other branches of knowledge. Yet not all subjects are equally amenable to formalization. Greek mathematics achieved its greatest successes in astronomy, where Ptolemy's calculations remained the standard of precision for centuries. But other subjects, such as medicine and law, depend more on deep experience than on brilliant mathematical calculations. Significantly, two of the most penetrating criticisms of logic were written by the physician Sextus Empiricus in the second century AD and by the legal scholar Ibn Taymiyya in the fourteenth century.

Sextus Empiricus, as his nickname suggests, was an empiricist. By profession, he was a physician; philosophically, he was an adherent of the school known as the Skeptics. Sextus maintained that all knowledge must come from experience. As an example, he cited the following syllogism:

Every human is an animal.

Socrates is human.

Therefore, Socrates is an animal.

Sextus admitted that this syllogism represents a valid inference pattern, but he questioned the source of evidence for the major premise *Every human is an animal*. A universal proposition that purports to cover every instance of some category must be derived by induction from particulars. If the induction is incomplete, then the universal proposition is not certain, and there might be some human who is not an animal. But if the induction is complete, then the particular instance Socrates must have been examimed already, and the syllogism is redundant or circular. Since every one of Aristotle's valid forms of syllogisms contains at least one universal affirmative or universal negative premise, the same criticisms apply to all of them: the conclusion must be either uncertain or circular.

The Aristotelians answered Sextus by claiming that universal propositions may be true by definition: since the type Human is defined as rational animal, the essence of human includes animal; therefore, no instance of human that was not an animal could exist. This line of defense was attacked by the Islamic jurist and legal scholar Taqi al-Din Ibn Taymiyya. Like Sextus, Ibn Taymiyya agreed that the form of a syllogism is valid, but he did not accept Aristotle's distinction between essence and accident (Hallaq 1993). According to Aristotle, the essence of human includes both rational and animal. Other attributes, such as laughing or being a featherless biped, might be unique to humans, but they are *accidental* attributes that could be different without changing the essence. Ibn Taymiyya, however, maintained that the distinction between essence and accident was arbitrary. Human might just as well be defined as laughing animal, with rational as an accidental attribute.

Denouncing logic would be pointless if no other method of reasoning were possible. But Ibn Taymiyya had an alternative: the legal practice of reasoning by cases and analogy. In Islamic law, a new case is *assimilated* to one or more previous cases that serve as precedents. The mechanism of assimilation is anal-ogy, but the analogy must be guided by a cause that is common to the new case as well as the earlier cases. If the same cause is present in all the cases, then the earlier judgment can be transferred to the new case. As an example, it is written in the Koran that grape wine is prohibited, but nothing is said about date wine. The judgment for date wine would be derived in four steps:

1. Given case: Grape wine is prohibited.

2. New case: Is date wine prohibited?

3. Cause: Grape wine is prohibited because it is intoxicating; date wine is also intoxicating.
4. Judgment: Date wine is also prohibited.

In practice, the reasoning may be more complex. Several previous cases may have a common cause but different judgments. Then the analysis must deter-mine whether there are mitigating circumstances that affect the operation of the cause. But the principles remain the same: analogy guided by rules of evi-dence and relevance determines the common cause, the effect of the mitigating circumstances, and the judgment.

Besides arguing in favor of analogy, Ibn Taymiyya also replied to the logicians who claimed that syllogistic reasoning is certain, but analogy is merely probable. He admitted that logical deduction is certain when applied to purely mental con-structions in mathematics. But in any reasoning about the real world, universal propositions can only be derived by induction, and induction must be guided by the same principles of evidence and relevance used in analogy. Figure 1 illus-trates Ibn Taymiyya's argument: Deduction proceeds from a *theory* containing universal propositions. But those propositions must have earlier been derived by induction

using the methods of analogy. The only difference is that induction produces a theory as intermediate result, which is then used in a subsequent process of deduction. By using analogy directly, legal reasoning dispenses with the intermediate theory and goes straight from cases to conclusion. If the theory and the analogy are based on the same evidence, they must lead to the same conclusions.

The question in Figure 1 represents some known aspects of a new case, which has unknown aspects to be determined. In deduction, the known aspects are com-pared (by a version of structure mapping called *unification*) with the premises of some implication. Then the unknown aspects, which answer the question, are derived from the conclusion of the implication. In analogy, the known aspects of the new case are compared with the corresponding aspects of the older cases. The case that gives the best match may be assumed as the best source of evidence.

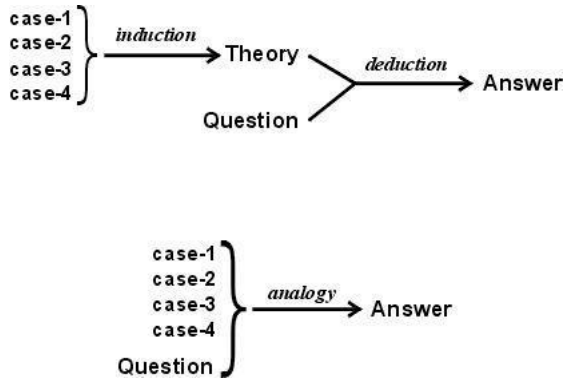



Fig. 1. Comparison of logical and analogical reasoning

for estimating the unknown aspects of the new case. The other cases show alter-native possibilities for those unknown aspects; the closer the agreement among the alternatives, the stronger the evidence for the conclusion.

Both Sextus Empiricus and Ibn Taymiyya admitted that logical reasoning is valid, but they doubted the source of evidence for universal propositions about the real world. What they overlooked was the pragmatic value of a good theory: a small group of scientists can derive a theory by induction, and anyone else can apply it without redoing the exhaustive analysis of cases. The two-step process of induction followed by deduction has proved to be most successful in the physical sciences, which include physics, chemistry, molecular biology, and the engineering practices they support. The one-step process of case-based reasoning, however, is more successful in fields

outside the so-called “hard” sciences, such as business, law, medicine, and psychology. Even in the “soft” sciences, which are rife with exceptions, a theory that is successful most of the time can still be useful. Many cases in law or medicine can be settled by the direct application of some general principle, and only the exceptions require an appeal to a long history of cases. And even in physics, the hardest of the hard sciences, the theories may be well established, but the question of which theory to apply to a given problem usually requires an application of analogy. In both science and daily life, there is no sharp dichotomy between subjects amenable to strict logic and those that require analogical reasoning.

The informal arguments illustrated in Figure 1 are supported by an analysis of the algorithms used for logical reasoning. Following is Peirce’s classification of the three kinds of logical reasoning and the way that the structure-mapping operations of analogy are used in each of them:

– Deduction. A typical rule used in deduction is *modus ponens*: given an assertion *p* and an axiom of the form *p* implies *q*, deduce the conclusion *q*. In most applications, the assertion *p* is not identical to the *p* in the axiom, and structure mapping is necessary to *unify* the two *p*s before the rule can be applied. The most time-consuming task is not the application of a single rule, but the repeated use of analogies for finding patterns that may lead to successful rule applications.

– Induction. When every instance of *p* is followed by an instance of *q*, induc-tion is peformed by assuming that *p* implies *q*. Since the *p*s and *q*s are rarely identical in every occurrence, a form of analogy called *generalization* is used to derive the most general implication that subsumes all the instances.

– Abduction. The operation of guessing or forming an initial hypothesis is what Peirce called abduction. Given an assertion *q* and an axiom of the form *p* implies *q*, the guess that *p* is a likely cause or explanation for *q* is an act of abduction. The operation of guessing *p* uses the least constrained version of analogy, in which some parts of the matching graphs may be more generalized while other parts are more specialized.

As this discussion indicates, analogy is a prerequisite for logical reasoning, which is a highly disciplined method of using repeated analogies. In both human reasoning and computer implementations, the same underlying operations can be used to support both.

## 3 Analogy Engine

The VivoMind Analogy Engine (VAE), which was developed by Majumdar, is a high-performance analogy finder that uses conceptual graphs for the knowledge representation. Like SME, structure mapping is used to find analogies. Unlike SME, the VAE algorithms can find analogies in time proportional to ($N \log N$), where $N$ is

the number of nodes in the current knowledge base or context. SME, however, requires time proportional to $N^3$ (Forbus et al. 1995). A later version called MAC/FAC reduced the time by using a search engine to extract the most likely data before using SME to find analogies (Forbus et al. 2002). With its greater speed, VAE can find analogies in the entire WordNet knowledge base in just a few seconds, even though WordNet contains over $10^5$ nodes. For that size, one second with an ($N \log N$) algorithm would correspond to 30 years with an $N^3$ algorithm.

VAE can process CGs from any source: natural languages, programming languages, and any kind of information that can be represented in graphs, such as organic molecules or electric-power grids. In an application to distributed inter-acting agents, VAE processes both English messages and signals from sensors that monitor the environment. To determine an agent's actions, VAE searches for analogies to what humans did in response to similar patterns of messages and signals. To find analogies, VAE uses three methods of comparison, which can be used separately or in combination:

1. Matching type labels. Method #1 compares nodes that have identical labels, labels that are related as subtype and supertype such as Cat and Animal, or labels that have a common supertype such as Cat and Dog.
2. Matching subgraphs. Method #2 compares subgraphs with possibly dif-ferent labels. This match succeeds when two graphs are isomorphic (independent of the labels) or when they can be made isomorphic by combining adjacent nodes.
3. Matching transformations. If the first two methods fail, Method #3 searches for transformations that can relate subgraphs of one graph to sub-graphs of the other.

These three methods of matching graphs were inspired by Peirce's categories of Firstness, Secondness, and Thirdness (Sowa 2000). The first compares two nodes by what they contain in themselves independent of any other nodes; the second compares nodes by their relationships to other nodes; and the third com-pares the mediating transformations that may be necessary to make the graphs comparable. To illustrate the first two methods, the following table shows an analogy found by VAE when comparing the background knowledge in WordNet for the concept types Cat and Car:

Analogy of Cat to Car

| Cat | Car |
|---|---|
| head | hood |
| eye | headlight |
| cornea | glass plate |
| mouth | fuel cap |
| stomach | fuel tank |
| bowel | combustion chamber |
| anus | exhaust pipe |

| | |
|---|---|
| skeleton | chasis |
| heart | engine |
| paw | wheel |
| fur | paint |

Fig. 2. An analogy discovered by VAE

As Figure 2 illustrates, there is an enormous amount of background knowl-edge stored in lexical resources such as WordNet. It is not organized in a form that is precise enough for deduction, but it is adequate for the more primitive method of analogy.

Since there are many possible paths through all the definitions and examples of WordNet, most comparisons generate multiple analogies. To evaluate the ev-idence for any particular mapping, a *weight of evidence* is computed by using heuristics that estimate the closeness of the match. For Method #1 of matching type labels, the closest match results from identical labels. If the labels are not identical, the weight of evidence decreases with the distance between the labels in the type hierarchy:

1. Identical type labels, such as Cat to Cat.

2. Subtype to supertype, such as Cat to Animal.
3. Siblings of the same supertype, such as Cat to Dog.
4. More distant cousins, such as Cat to Tree.

For Method #2 of matching subgraphs, the closest match results from find-ing that both graphs, in their entirety, are isomorphic. The weight of evidence decreases as their common subgraphs become smaller or if the graphs have to be modified in order to force a match:

1. Match isomorphic graphs.

2. Match two graphs that have isomorphic subgraphs (the larger the subgraphs, the stronger the evidence for the match).

3. Combine adjacent nodes to make the subgraphs isomorphic.

The analogy shown in Figure 2 received a high weight of evidence because VAE found many matching labels and large matching subgraphs in correspond-ing parts of a cat and parts of a car:

– Some of the corresponding parts have similar functions: fur and paint are outer coverings; heart and engine are internal parts that have a regular beat; skeleton and chasis are structures to which other parts are attached; paw and wheel perform a similar function, and there are four of each.

– The longest matching subgraph is the path from mouth to stomach to bowel to anus of a cat, which matches the path from fuel cap to fuel tank to combustion chamber to exhaust pipe of a car. The stomach of a cat and the fuel tank of a car are analogous because they are both subtypes of Container. The bowel and the combustion chamber perform analogous functions. The mouth and the fuel cap are considered input orifices, and the anus and the exhaust pipe are outputs. The weight of evidence is somewhat reduced because adjustments must be made to ignore nodes that do not match: the esophagus of a cat does not match anything in WordNet's description of a car, and the muffler of a car does not match anything in its description of a cat.

– A shorter subgraph is the path from head to eyes to cornea of a cat, which matches the path from hood to headlights to glass plate of a car. The head and the hood are both in the front. The eyes are analogous to the headlights because there are two of each and they are related to light, even though the relationships are different. The cornea and the glass plate are in the front, and they are both transparent.

Each matching label and each structural correspondence contributes to the weight of evidence for the analogy, depending on the closeness of the match and the exactness of the correspondence.

As the cat-car comparison illustrates, analogy is a versatile method for using informal, unstructured background knowledge. But analogies are also valuable for comparing the highly formalized knowledge of one axiomatized theory to another. In the process of theory revision, Niels Bohr used an analogy between gravitational force and electrical force to derive a theory of the hydrogen atom as analogous to the earth revolving around the sun. Method #3 of analogy, which finds matching transformations, can also be used to determine the precise mappings required for transforming one theory or representation into another.

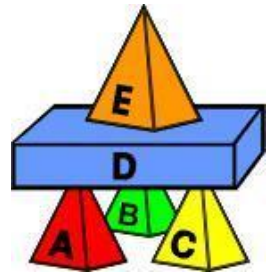
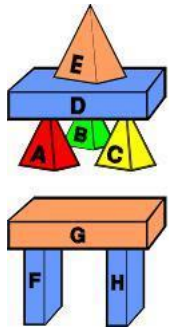



Fig. 3. A physical structure to be represented by data

As an example, Figure 3 shows a physical structure that could be represented by many different data structures.

Programmers who use different tools, databases, or programming languages often use different, but analogous representations for the same kinds of infor-mation. LISP programmers, for example, prefer to use lists, while FORTRAN programmers prefer vectors. Conceptual graphs are a highly general representa-tion, which can represent any kind of data stored in a digital computer, but the types of concepts and relations usually reflect the choices made by the original programmer, which in turn reflect the options available in the original program-ming tools. Figure 4 shows a representation for Figure 3 that illustrates the typical choices used with relational databases.




Objects

| ID | Shape | Color |
|---|---|---|
| A | pyramid | red |
| B | pyramid | green |
| C | pyramid | yellow |
| D | block | blue |
| E | pyramid | orange |
| F | block | blue |
| G | block | orange |
| H | block | blue |

Supports

| Supporter | Supportee |
|---|---|
| A | D |
| B | D |
| C | D |
| D | E |
| F | G |
| H | G |

Fig. 4. Two structures represented in a relational database

On the left of Figure 4 are two structures: a copy of Figure 3 and an arch con-structed from three blocks. On the right are two tables: the one labeled Objects lists the identifiers of all the objects in both tables with their shapes and colors; the one labeled Supports lists each object that supports (labeled Supporter) and the object supported (labeled Supportee). As Figure 4 illustrates, a rela-tional database typically scatters the information about a single object or struc-ture of objects into multiple tables. For the structure of pyramids and blocks, each object is listed once in the Objects table, and one or more times in either or both columns of the Supports table. Furthermore, information about the two disconnected structures shown on the left is intermixed in both tables. When all the information about the structure at the top left is extracted from both tables of Figure 4, it can be mapped to the conceptual graph of Figure 5.

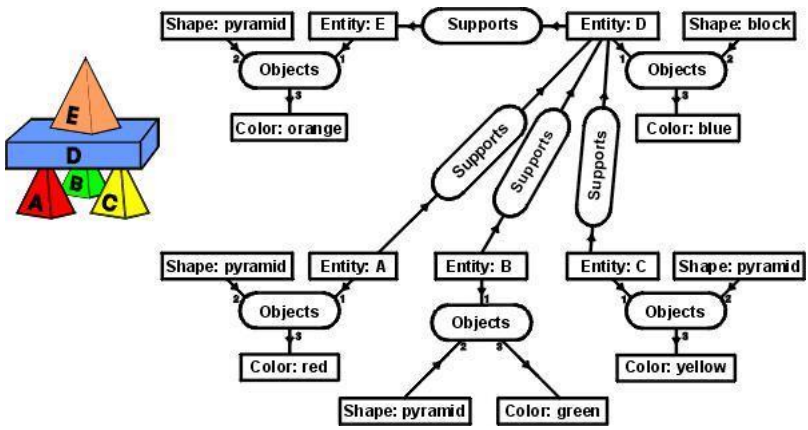



Fig. 5. A CG derived from the relational DB

In Figure 5, each row of the table labeled Objects is represented by a con-ceptual relation labeled Objects, and each row of the table labeled Supports is represented by a conceptual relation labeled Supports. The type labels of the concepts are mostly derived from the labels on the columns of the two tables in Figure 4. The only exception is the label Entity, which is used instead of ID. The reason for that exception is that ID is a metalevel term about the rep-resentation language; it is not a term that is derived from the entities in the domain of discourse. The concept [Entity: E], for example, says that E is an instance of type Entity. The concept [ID: "E"], however, would say that the character string "E" is an instance of type ID. The use of the label Entity instead of ID avoids mixing the metalevel with the object level. Such mixing of levels is common in most programs, since the computer ignores any meaning that might be associated with the labels. In logic, however, the fine distinctions are important, and CGs mark them consistently.

When natural languages are translated to CGs, the distinctions must be en-forced by the semantic interpreter. Figure 6 shows a CG that represents the English sentence, *A red pyramid A, a green pyramid B, and a yellow pyramid C support a blue block D, which supports an orange pyramid E.* The conceptual re-lations labeled Thme and Inst represent the case relations theme and instrument. The relations labeled Attr represent the attribute relation between a concept of some entity and a concept of some attribute of that entity. The type labels.

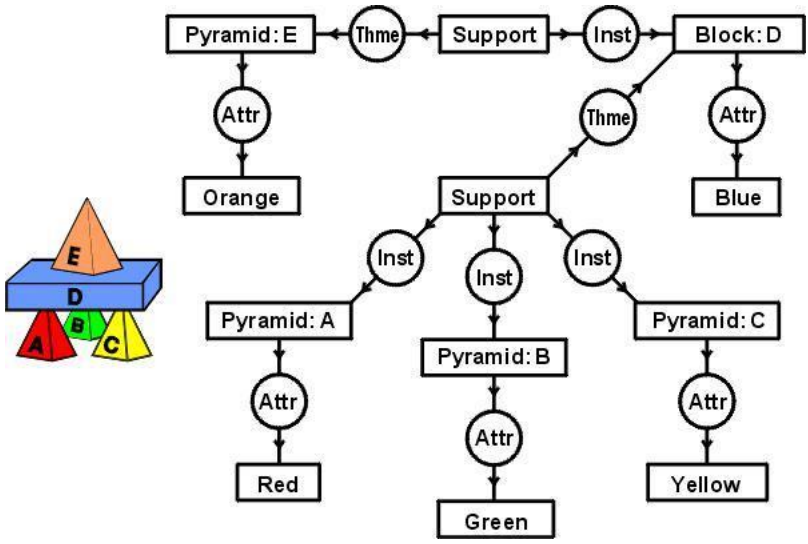



Fig. 6. A CG derived from an English sentence

of concepts are usually derived from nouns, verbs, adjectives, and adverbs in English.

Although the two conceptual graphs represent equivalent information, they look very different. In Figure 5, the CG derived from the relational database has 15 concept nodes and 9 relation nodes. In Figure 6, the CG derived from English has 12 concept nodes and 11 relation nodes. Furthermore, no type label on any node in Figure 5 is identical to any type label on any node in Figure 6. Even though some character strings are similar, their positions in the graphs cause them to be treated as distinct. In Figure 5, orange is the name of an instance of type Color; and in Figure 6, Orange is the label of a concept type. In Figure 5, Supports is the label of a relation type; and in Figure 6, Support is not only the label of a concept type, it also lacks the final S.

Because of these differences, the strict method of unification cannot show that the graphs are identical or even related. Even the more relaxed methods of matching labels or matching subgraphs are unable to show that the two graphs are analogous. Method #3 of analogy, however, can find matching transforma-tions that can translate Figure 5 into Figure 6 or vice-versa. When VAE was asked to compare those two graphs, it found the two transformations shown in Figure 7. Each transformation determines a mapping between a type of subgraph in Figure 5 and another type of subgraph in Figure 6.

The two transformations shown in Figure 7 define a version of *graph grammar* for parsing one kind of graph and mapping it to the other. The transformation at the top of Figure 7 can be applied to the five subgraphs containing the relations of type Objects in Figure 5 and relate them to the five subgraphs containing the relations of type Attr in Figure 6. That same transformation could be applied in

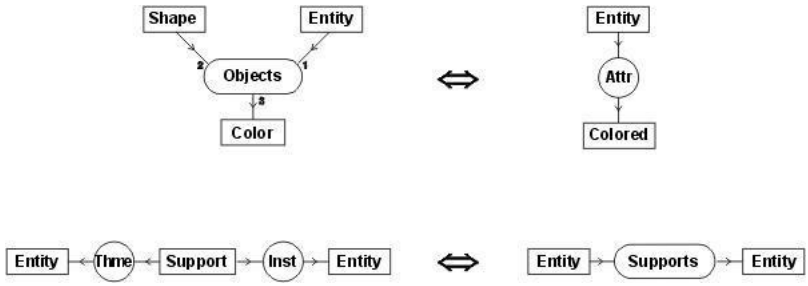



Fig. 7. Two transformations discovered by VAE

reverse to relate the five subgraphs of Figure 6 to the five subgraphs of Figure 5. The transformation at the bottom of Figure 7 could be applied from right to left in order to map Figure 6 to Figure 5. When applied in that direction, it would map three different subgraphs, which happen to contain three common nodes: the subgraph extending from [Pyramid: A] to [Block: D]; the one from [Pyramid: B] to [Block: D]; and the one from [Pyramid: C] to [Block: D]. When applied in the reverse direction, it would map three subgraphs of Figure 5 that contained only one common node.

The transformations shown in Figure 7 have a high weight of evidence be-cause they are used repeatedly in exactly the same way. A single transformation of one subgraph to another subgraph with no matching labels would not con-tribute anything to the weight of evidence. But if the same transformation is applied twice, then its likelihood is greatly increased. Transformations that can be applied three times or five times to relate all the nodes of one graph to all the nodes of another graph have a likelihood that comes close to being a certainty.

Of the three methods of analogy used in VAE, the first two – matching labels and matching subgraphs – are also used in SME. Method #3 of matching transformations, which only VAE is capable of performing, is more complex because it depends on analogies of analogies. Unlike the first two methods, which VAE can perform in ($N$ log $N$ ) time, Method #3 takes polynomial time, and it can only be applied to much smaller amounts of data. In practice, Method #3 is usually applied to small parts of an

analogy in which most of the mapping is done by the first two methods and only a small residue of unmatched nodes remains to be mapped. In such cases, the number $N$ is small, and the mapping can be done quickly. Even for mapping Figure 5 (with $N$ = 9) to Figure 6 (with $N$ = 11), the Method #3 took a few seconds, whereas the time for Methods #1 and #2 on graphs of such size would be less than a millisecond.

Each of the three methods of analogy determines a mapping of one CG to another. The first two methods determine a node-by-node mapping of CGs, where some or all of the nodes of the first CG may have different type labels from the corresponding nodes of the other. Method #3 determines a more complex map-ping, which comprises multiple mappings of subgraphs of one CG to subgraphs of the other. These methods can be applied to CGs derived from any source, including natural languages, logic, or programming languages.

In one major application, VAE was used to analyze the programs and documentation of a large corporation, which had systems in daily use that were up to forty years old (LeClerc & Majumdar 2002). Although the documentation specified how the programs were supposed to work, nobody knew what errors, discrepancies, and obsolete business procedures might be buried in the code. The task required an analysis of 100 megabytes of English, 1.5 million lines of COBOL programs, and several hundred control-language scripts, which called the programs and specified the data files and formats. Over time, the English terminology, computer formats, and file names had changed. Some of the for-mat changes were caused by new computer systems and business practices, and others were required by different versions of federal regulations. In three weeks of computation on a 750 MHz Pentium III, VAE combined with the Intellitex parser was able to analyze the documentation and programs, translate all state-ments that referred to files, data, or processes in any of the three languages (English, COBOL, and JCL) to conceptual graphs, and use the CGs to generate an English glossary of all processes and data, to define the specifications for a data dictionary, to create dataflow diagrams of all processes, and to detect inconsistencies between the documentation and the implementation.

## 4 Inference Engine

Most theorem provers use a tightly constrained version of structure mapping called *unification*, which forces two structures to become identical. Relaxing constaints in one direction converts unification to generalization, and relaxing them in another direction leads to specialization. With arbitrary combinations of generalization and specialization, there is a looser kind of similarity, which, if there is no limit on the extent, could map any graph to any other. When Peirce's rules of inference are redefined in terms of generalization and specialization, they support an inference procedure that can use exactly the same algorithms and data structures designed for

the VivoMind Analogy Engine. The primary difference between the analogy engine and the inference engine is in the strategy that schedules the algorithms and determines which constraints to enforce.

When Peirce invented the implication operator for Boolean algebra, he ob-served that the truth value of the antecedent is always less than or equal to the truth value of the consequent. Therefore, the symbol $\preceq$ may be used to repre-sent implication: $p \preceq q$ means that the truth value of $p$ is less than or equal to the truth value of $q$. That same symbol may be used for generalization: if a graph or formula $p$ is true in fewer cases than another graph or formula $q$, then $p$ is more specialized and $q$ is more generalized. Figure 8 shows a generalization hierarchy in which the most general CG is at the top. Each dark line in Figure 8 represents the $\preceq$ operator: the CG above is a generalization, and the CG below is a specialization.

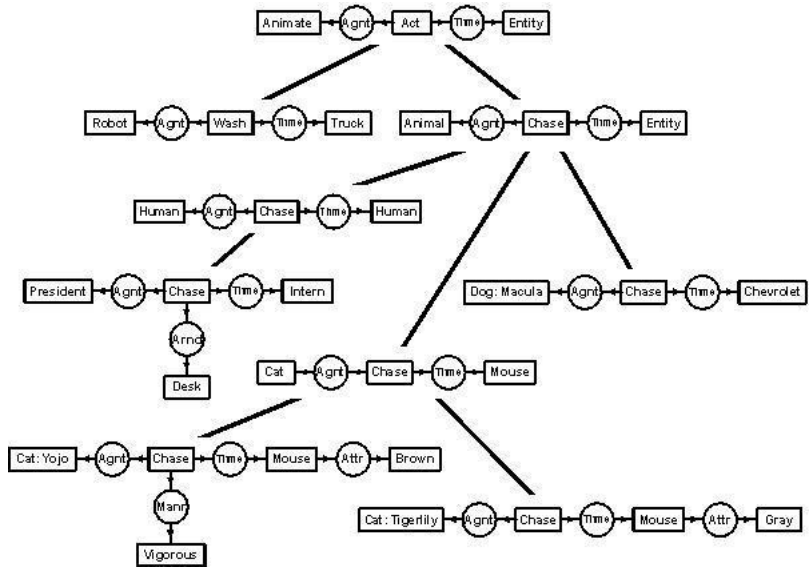



Fig. 8. A generalization hierarchy of CGs

The top CG says that an animate being is the agent of some act that has an entity as the theme of the act. Below it are two specializations: a CG for a robot washing a truck, and a CG for an animal chasing an entity. The CG for an animal chasing an entity has three specializations: a human chasing a human, a cat chasing a mouse, and the dog Macula chasing a Chevrolet. The two graphs at the bottom represent the most specialized sentences: *The cat Yojo is vigorously chasing a brown mouse*, and *the cat Tigerlily is chasing a gray mouse*.

The operations on conceptual graphs are based on combinations of six *canon-ical formation rules*, which perform the structure-building operations of percep-tion and the structure-mapping operations of analogy. Logically, each rule has one of three possible effects on a CG: the rule can make it more specialized, more generalized, or logically equivalent but with a modified shape. Each rule has an inverse rule that restores a CG to its original form. The inverse of specialization is generalization, the inverse of generalization is specialization, and the inverse of equivalence is another equivalence.

All the graphs in Figure 8 belong to the *existential-conjunctive* subset of logic, whose only operators are the existential quantifier ∃ and the conjunction ∧. For this subset, the canonical formation rules take the forms illustrated in Figures 5, 6, and 7. These rules are fundamentally graphical: they are easier to show than to describe. Sowa (2000) presented the formal definitions, which specify the details of how the nodes and arcs are affected by each rule.

Figure 9 shows the first two rules: *copy* and *simplify*. At the top is a CG for the sentence "The cat Yojo is chasing a mouse." The down arrow represents two applications of the copy rule. The first copies the Agnt relation, and the second

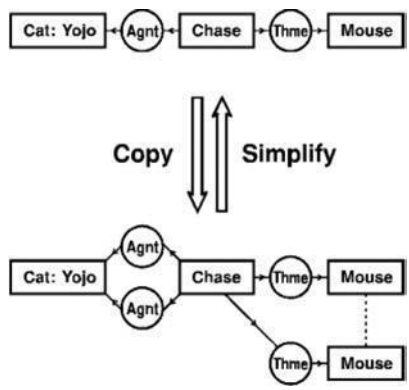



Fig. 9. Copy and simplify rules

copies the subgraph →(Thme)→[Mouse]. The two copies of the concept [Mouse] at the bottom of Figure 9 are connected by a dotted line called a *coreference link*; that link, which corresponds to an equal sign = in predicate calculus, indicates that both concepts must refer to the same individual. Since the new copies do not add any information, they may be erased without losing information. The up arrow represents the simplify rule, which performs the inverse operation of erasing redundant copies. The copy and simplify rules are called *equivalence rules* because any two CGs that can be transformed from one to the other by any combination of copy and simplify rules are logically equivalent. The two formulas in predicate calculus that are derived

from Figure 9 are also logically equivalent. The top CG maps to the following formula:

$$(\exists x{:}Cat)(\exists y{:}Chase)(\exists z{:}Mouse)(name(x,'Yojo') \wedge agnt(y,x) \wedge thme(y,z)),$$

In the formula that corresponds to the bottom CG, the equality $z=w$ repre-sents the coreference link that connects the two copies of [Mouse]:

$$(\exists x{:}Cat)(\exists y{:}Chase)(\exists z{:}Mouse)(\exists w{:}Mouse)(name(x,'Yojo') \wedge agnt(y,x)$$

$$\wedge agnt(y,x) \wedge thme(y,z) \wedge thme(y,w) \wedge z=w).$$

By the inference rules of predicate calculus, either of these two formulas can be derived from the other.

Figure 10 illustrates the *restrict* and *unrestrict* rules. At the top is a CG for the sentence "A cat is chasing an animal." Two applications of the restrict rule transform it to the CG for "The cat Yojo is chasing a mouse." The first step is a *restriction by referent* of the concept [Cat], which represents some indefinite cat, to the more specific concept [Cat: Yojo], which represents a particular cat named Yojo. The second step is a *restriction by type* of the concept [Animal] to a concept of the subtype [Mouse]. Two applications of the unrestrict rule perform the inverse transformation of the bottom graph to the top graph. The restrict rule is a *specialization rule*, and the unrestrict rule is a *generalization rule*. The

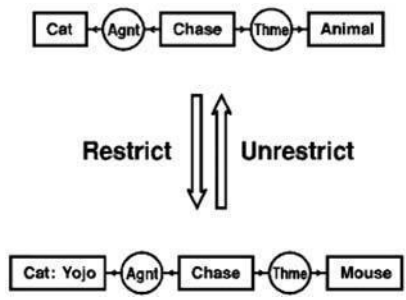



Fig. 10. Restrict and unrestrict rules

more specialized graph implies the more general one: if the cat Yojo is chasing a mouse, it follows that a cat is chasing an animal. The same implication holds for the corresponding formulas in predicate calculus. The more general formula

$(\exists x{:}Cat)(\exists y{:}Chase)(\exists z{:}Animal)(agnt(y,x) \wedge thme(y,z))$

is implied by the more specialized formula

$(\exists x{:}Cat)(\exists y{:}Chase)(\exists z{:}Mouse)(name(x,'Yojo') \wedge agnt(y,x) \wedge thme(y,z))$.

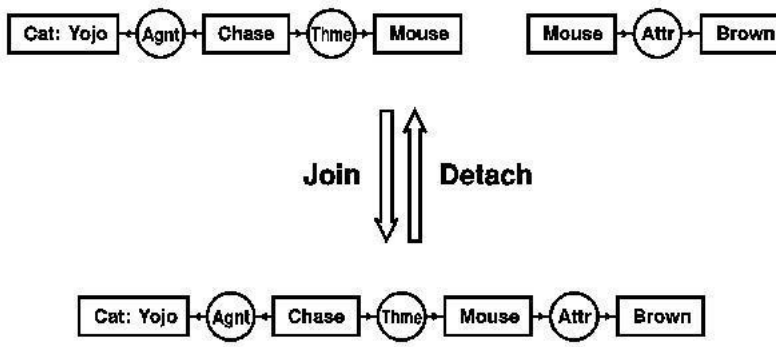



Fig. 11. Join and detach rules

Figure 11 illustrates the *join* and *detach* rules. At the top are two CGs for the sentences "Yojo is chasing a mouse" and "A mouse is brown." The join rule overlays the two identical copies of the concept [Mouse], to form a single CG for the sentence "Yojo is chasing a brown mouse." The detach rule performs the inverse operation. The result of join is a more specialized graph that implies the one derived by detach. The same implication holds for the corresponding formulas in predicate calculus. The conjunction of the formulas for the top two CGs

$(\exists x{:}Cat)(\exists y{:}Chase)(\exists z{:}Mouse)(name(x,'Yojo') \wedge agnt(y,x) \wedge thme(y,z))$

$\wedge (\exists w{:}Mouse)(\exists v{:}Brown)attr(w,v)$ $(\exists x{:}Cat)(\exists y{:}Chase)(\exists z{:}Mouse)(\exists v{:}Brown)(name(x,'Yojo')$

is implied by the formula for the bottom CG

$(\exists x{:}Cat)(\exists y{:}Chase)(\exists z{:}Mouse)(\exists v{:}Brown)(name(x,'Yojo') \wedge agnt(y,x)$

∧ thme(y,z) ∧ attr(z,v)).

These rules can be applied to full first-order logic by specifying how they interact with negation. In CGs, each negation is represented by a context that has an attached relation of type Neg or its abbreviation by the symbol ¬ or ~. A *positive context* is nested in an even number of negations (possibly zero). A *negative context* is nested in an odd number of negations. The following four principles determine how negations affect the rules:

1. *Equivalence rules*. An equivalence rule remains an equivalence rule in any context, positive or negative.
2. *Specialization rules*. In a negative context, a specialization rule becomes a generalization rule; but in a positive context, it remains a specialization rule.
3. *Generalization rules*. In a negative context, a generalization rule becomes a specialization rule; but in a positive context, it remains a generalization rule.
4. *Double negation*. A *double negation* is a nest of two negations in which no concept or relation node occurs between the inner and the outer negation. (It is permissibe for an arc of a relation or a coreference link to cross the space between the two negations, but only if one endpoint is inside the inner negation and the other endpoint is outside the outer negation.) Then drawing or erasing a double negation around any CG or any subgraph of a CG is an equivalence operation.

In short, a single negation reverses the effect of generalization and special-ization rules, but it has no effect on equivalence rules. Since drawing or erasing a double negation adds or subtracts two negations, it has no effect on any rule.

By handling the syntactic details of conceptual graphs, the canonical forma-tion rules enable the rules of inference to be stated in a form that is independent of the graph notation. For each of the six rules, there is an equivalent rule for predicate calculus or any other notation for classical FOL. To derive equiva-lent rules for other notations, start by showing the effect of each rule on the existential-conjunctive subset (no operators other than ∃ and ∧ ). To handle negation, add one to the negation count for each subgraph or subformula that is governed by a ~ symbol. For other operators ( ∀, ⊃ , and ∨ ), count the number of negations in their definitions. For example, $p \supset q$ is defined as $\sim(p \wedge \sim q)$; therefore, the subformula $p$ is nested inside one additional negation, and the subformula $q$ is nested inside two additional negations.

When the CG rules are applied to other notations, some extensions may be necessary. For example, the *blank* or empty graph is a well-formed EG or CG, which is always true. In predicate calculus, the blank may be represented by

a constant formula T, which is defined to be true. The operation of erasing a graph would correspond to replacing a formula by T. When formulas are erased or inserted, an accompanying conjunction symbol must also be erased or inserted in some notations. Other notations, such as the Knowledge Interchange Format (KIF), are closer to CGs because they only require one conjunction symbol for an arbitrarily long list of conjuncts. In KIF, the formula (and), which is an empty list of conjuncts, may be used as a synonym for the blank graph or T. Discourse representation structures (DRSs) are even closer to EGs and CGs because they do not use any symbol for conjunction; therefore, the blank may be considered a DRS that is always true.

Peirce's rules, which he stated in terms of existential graphs, form a sound and complete system of inference for first-order logic with equality. If the word *graph* is considered a synonym for *formula* or *statement*, the following adaptation of Peirce's rules can be applied to any notation for FOL, including EGs, CGs, DRS, KIF, or the many variations of predicate calculus. These rules can also be applied to subsets of FOL, such as description logics and Horn-clause rules.

– *Erasure*. In a positive context, any graph or subgraph *u* may be replaced by a generalization of *u*; in particular, *u* may be erased (i.e. replaced by the blank, which is a generalization of every graph).

– *Insertion*. In a negative context, any graph or subgraph *u* may be replaced by a specialization of *u*; in particular, any graph may be inserted (i.e. it may replace the blank).

– *Iteration*. If a graph or subgraph *u* occurs in a context C, another copy of *u* may be inserted in the same context C or in any context nested in C.

– *Deiteration*. Any graph or subgraph *u* that could have been derived by iter-ation may be erased.

– *Equivalence*. Any equivalence rule (copy, simplify, or double negation) may be performed on any graph or subgraph in any context.

These rules, which Peirce formulated in several equivalent variants from 1897 to 1909, form an elegant and powerful generalization of the rules of *natural de-duction* by Gentzen (1935). Like Gentzen's version, the only axiom is the blank. What makes Peirce's rules more powerful is the option of applying them in any context nested arbitrarily deep. That option shortens many proofs, and it elim-inates Gentzen's bookkeeping for making and discharging assumptions. For fur-ther discussion and comparison, see MS 514 (Peirce 1909) and the commentary that shows how other rules of inference can be derived from Peirce's rules.

Unlike most proof procedures, which are tightly bound to a particular syntax, this version of Peirce's rules is stated in notation-independent terms of gener-alization and specialization. In this form, they can even be applied to natural languages. For any language, the first step is to show how each syntax rule affects generalization, specialization, and equivalence. In counting the negation depth for natural languages, it is important to recognize the large number of negation words, such as *not, never, none, nothing, nobody*, or *nowhere*. But many other words also contain implicit negations, which affect any context governed by those words. Verbs like *prevent* or *deny*, for example, introduce a negation into any clause or phrase in their complement. Many adjectives also have implicit nega-tions: a stuffed bear, for example, lacks essential properties of a bear, such as being alive. After the effects of these features on generalization and specialization have been taken into account in the syntactic definition of the language, Peirce's rules can be applied to a natural language as easily as to a formal language.

## 5 A Semeiotic Foundation for Cognition

Peirce developed his theory of signs or *semeiotic* as a species-independent theory of cognition. He considered it true of any "scientific intelligence," by which he meant any intelligence that is "capable of learning from experience." Peirce was familiar with Babbage's mechanical computer; he was the first person to suggest that such machines should be based on electical circuits rather than mechani-cal linkages; and in 1887, he published an article on "logical machines" in the *American Journal of Psychology*, which even today would be a respectable com-mentary on the possibilities and difficulties of artificial intelligence. His definition of *sign* is independent of any implementation in proteins or silicon:

> I define a sign as something, A, which brings something, B, its interpre-tant, into the same sort of correspondence with something, C, its object, as that in which itself stands to C. In this definition I make no more reference to anything like the human mind than I do when I define a line as the place within which a particle lies during a lapse of time. (1902, p. 235)

As a close friend of William James, Peirce was familiar with the experimental psychology of his day, which he considered a valuable study of what possibilities are realized in any particular species. But he considered semeiotic to be a more fundamental, implementation-independent characterization of cognition.

Peirce defined logic as "the study of the formal laws of signs" (1902, p. 235), which implies that it is based on the same kinds of semeiotic operations as all other cognitive processes. He reserved the word *analogy* for what is called "ana-logical reasoning" in this paper. For the kinds of structure mapping performed by SME and VAE, Peirce used the term *diagrammatic reasoning*, which he de-scribed as follows:

> The first things I found out were that all mathematical reasoning is di-agrammatic and that all necessary reasoning is mathematical reasoning, no matter how simple it may be. By diagrammatic reasoning, I mean reasoning which constructs a diagram according to a precept expressed in general terms, performs experiments upon this diagram, notes their results, assures itself that similar experiments performed upon any di-agram constructed according to the same precept would have the same results, and expresses this in general terms. This was a discovery of no little importance, showing, as it does, that all knowledge without excep-tion comes from observation. (1902, pp. 91-92)

This short paragraph summarizes many themes that Peirce developed in more detail in his other works. In fact, it summarizes the major themes of this article:

1. By a diagram, Peirce meant any abstract pattern of signs. He included the patterns of algebra and Aristotle's syllogisms as diagrammatic, but he also said that his existential graphs were "more diagrammatic".

2. His "experiments" upon a diagram correspond to various AI procedures, such as generate and test, backtracking, breadth-first parallel search, and path following algorithms, all of which are performed on data structures that correspond to Peirce's notion of "diagram".

3. The first sentence of the paragraph applies the term "mathematical" to any kind of deduction, including Aristotle's syllogisms and any informal logic that may be used in a casual conversation.

4. The last sentence, which relates observation to diagrammatic reasoning, echoes the themes of the first section of this paper, which emphasized the need for an integration of of perception with the mechanisms of analogy. Peirce stated that point even more forcefully: "Nothing unknown can ever become known except through its analogy with other things known" (1902, p. 287).

In short, the operations of diagrammatic reasoning or structure mapping form the bridge from perception to all forms of reasoning, ranging from the most casual to the most advanced. Forbus et al. (2002) applied the term *reasoning from first principles* to logic, not to analogical reasoning. But Peirce, who invented two of the most widely used notations for logic, recognized that the underlying semeiotic mechanisms were more fundamental. The VivoMind implementation confirms Peirce's intuitions.

For natural language understanding, the constrained operations of unification and generalization are important, but exceptions, metaphors, ellipses, novel word senses, and the inevitable errors require less constrained analogies. When the VAE algorithms

are used in the semantic interpreter, there are no exceptions: analogies are used at every step, and the only difference between unification, generalization, and looser similarities is the nature of the constraints on the analogy.

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